2777. Proposed by W. D. CAIRNS, Oberlin College.

Prove that the two series

$$1 + \frac{\pi^4}{2^4 \cdot 4!} + \frac{\pi^8}{2^8 \cdot 8!} + \cdots,$$

and

$$\frac{\pi^2}{2^2 \cdot 2!} + \frac{\pi^6}{2^6 \cdot 6!} + \frac{\pi^{10}}{2^{10} \cdot 10!} + \cdots$$

are equal.

2778. Proposed by WARREN WEAVER, University of Wisconsin.

A partition of space is effected by means of five planes, none of which are parallel and no four of which pass through the same point, and six spheres. This divides all space into n regions, some of which are finite and some infinite. Considering it equally probable that a bird be in any one of the n regions show that the probability of its being caught (that is, of its being in one of the finite regions) is equal to or less than 78/99.

2779. Proposed by J. L. RILEY, Junior Agricultural and Mechanics College, Stephenville, Texas.

A parabola is placed with its axis horizontal; find the straight line of shortest descent from the curve to the focus.

406 (Algebra) [March, 1914]. Proposed by S. A. COREY, Albia, Iowa.

Solve the system of equations:

$$(1 - x)(a_1 + a_2y + a_3z) = d, \quad (1 - y)(b_1 + b_2x + b_3z) = g, \quad (1 - z)(c_1 + c_2x + c_3y) = h.$$

411 (Algebra) [April, 1914]. Proposed by V. M. SPUNAR, Chicago, Ill.

Determine $x_1, x_2, x_3, \cdots x_p$, from the equations:

$$\begin{aligned} x_1 + x_2 + x_3 + \cdots + x_p &= a_0, \\ b_1x_1 + b_2x_2 + b_3x_3 + \cdots + b_px_p &= a_1, \\ b_1^2x_1 + b_2^2x_2 + b_3^2x_3 + \cdots + b_p^2x_p &= a_2, \\ \cdots \cdots \cdots \cdots \cdots \cdots & \cdots \\ b_1^{p-1}x_1 + b_2^{p-1}x_2 + b_3^{p-1}x_3 + \cdots + b_p^{p-1}x_p &= a_{p-1}. \end{aligned}$$

442 (Geometry) [May, 1914]. Proposed by J. B. SMITH, Hampden-Sidney College.

If any three straight lines AD, BE, CF, be drawn from the corners of the triangle ABC to the opposite sides a, b, c, they will enclose an area. If Δ, Δ'' be the areas of the triangles ABC, DEF, show that

$$\frac{\Delta''}{\Delta} = \frac{(AF \cdot BD \cdot CE - AE \cdot CD \cdot BF)^2}{(ab - CE \cdot CD)(bc - AE \cdot AF)(ca - BF \cdot BD)},$$

where the signs of the factors are to be determined by the following rule: Each segment being measured from one of the corners of the triangle ABC, along one of the sides, is to be regarded as positive or negative according as it is drawn towards or from the other corner in that side.

455 (Geometry) [February, 1915]. Proposed by R. P. BAKER, University of Iowa.

Find the minimum triangle of assigned angles inscribed in a given triangle.

348 (Calculus) [December, 1913]. Proposed by E. L. DODD, University of Texas.

Let $(x_1, x_2, \cdots x_n)$ be a point in n dimensions lying in the "sphere" S defined by

$$x_1^2 + x_2^2 + \cdots + x_n^2 \leqq 1.$$

Let T be that part of S defined by a set of n linear homogeneous inequalities with non-vanishing determinant; thus:

$$a_ix_1 + b_ix_2 + \cdots + k_ix_n \geqq 0, \quad i = 1, 2, \cdots n.$$

Find the value of

$$\frac{\int \cdot \underset{T}{\cdot} \cdot \int dx_1 \cdots dx_n}{\int \cdot \underset{S}{\cdot} \cdot \int dx_1 \cdots dx_n};$$

in other words, find the magnitude of a "solid angle" in n dimensions, with the "sphere" as unit solid angle.

Note. This problem was discussed and left unsolved by Schläfli in the *Quarterly Journal of Mathematics* for 1858, 1860, 1867.—Editor.

349 (Calculus) [December, 1913]. Proposed by C. N. SCHMALL, New York City.

If $y = a \cos (\log x) + b \sin (\log x)$, eliminate the constants a and b and obtain the equation

$$x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0.$$

198 (Number Theory) [November, 1913]. Proposed by the late ARTEMAS MARTIN.

Prove that every even number is the sum of two prime numbers.

Note. This problem has long been known and no proof has ever been given.—Editor.

201 (Number Theory) [December, 1913]. Proposed by E. T. BELL, University of Washington.

Eisenstein proposed (*Crelle*, t. 27, p. 282), as the simplest of several problems: "In the expansion of

$$\frac{1 + z + z^2 + \cdots + z^{p-1}}{(1 - z)^{p-1}} - 1,$$

where p is prime, to show that the coefficients of the various powers of z are all divisible by p."

SOLUTIONS OF PROBLEMS.

231 (Number Theory) [April, 1915]. Proposed by A. J. KEMPNER, University of Illinois.

Is the series whose terms are the reciprocals of all positive integers not containing a given combination of figures, for example not containing the combination 37, convergent or divergent? Numbers such as $\frac{1}{37}$, $\frac{1}{370}$, $\frac{1}{5371}$ shall be omitted, numbers such as $\frac{1}{73}$, $\frac{1}{307}$, $\frac{1}{5317}$ shall be admitted as terms of the series. (Compare AMERICAN MATHEMATICAL MONTHLY, Volume 21, page 123.)

SOLUTION BY FRANK IRWIN, University of California.

Let $f(n)$ be the number of numbers in the class, g_n, of numbers of n digits that do not contain the given combination, for example 37. We can get these numbers by taking any number of g_{n-1} and adding a digit at the end; except that this digit must not be 7 if the number chosen from g_{n-1} ends with a 3. Since the number of these rejected numbers of g_{n-1} is evidently $f(n - 2)$, we have the formula: $f(n) = 10f(n - 1) - f(n - 2)$. In the general case, where the given combination consists of k digits, we should get similarly, for $n > k$,

$$f(n) = 10f(n - 1) - f(n - k). \tag{1}$$

(An exception would arise for such a combination as 37537; here, for instance, the formula $f(8) = 10f(7) - f(3)$ would not hold, for the number 375 belonging to g_3 would have been subtracted, as it should not have been, since the number obtained by writing after it 3753, viz., 3753753, does not belong to g_7, and so has not been counted in $f(7)$. It will be seen that such cases arise when the first l digits of the given combination are the same as the last l, $l < k$. We shall call such combinations "improper" and deal with them separately.)

From (1) we have $f(n - 1) > f(n)/10$. Also since $f(n - 1) = f(n)/10$, $n < k$, and $f(n - 1) \geqq f(n)/10$, $n = k$ ($=$ if the given combination begins with a zero), we have, for $n > k$, the string of relations $f(n - 2) \geqq f(n - 1)/10$, $f(n - 3) \geqq f(n - 2)/10$, $\cdots$. From these last inequalities we derive $f(n - k) \geqq f(n - 1)/10^{k-1}$. Comparing this with (1), we see that $f(n) \leqq 10f(n - 1) - f(n - 1)/10^{k-1}$, or $f(n) \leqq 10rf(n - 1)$, if we put $(10^k - 1)/10^k = r$. Since the reciprocal of any number in g_n is $\leqq 1/10^{n-1}$, it follows that the sum of the reciprocals of the numbers in g_k, g_{k+1}, $\cdots$ are less than the corresponding terms of the series